Localization of electromagnetic waves in two-dimensional random dielectric systems

Bikash C. Gupta, Yu-Yu Chen, and Zhen Ye*

Wave Phenomena Laboratory, Department of Physics,
National Central University, Chungli, Taiwan 32054
(Dated: January 14, 2003)

We rigorously calculate the propagation and scattering of electromagnetic waves by rectangular and random arrays of dielectric cylinders in a uniform medium. For regular arrays, the band structures are computed and complete bandgaps are discovered. For random arrays, the phenomenon of wave localization is investigated and compared in two scenarios: (1) wave propagating through the array of cylinders; this is the scenario which has been commonly considered in the literature, and (2) wave transmitted from a source located inside the ensemble. We show that within complete band gaps, results from the two scenarios are similar. Outside the gaps, however, there is a distinct fundamental difference, that is, waves can be blocked from propagation by disorders in the first scenario, but such an inhibition may not lead to inhibition or wave localization in the second scenario. The study suggests that the traditional method may be ambiguous in discerning localization effects.

Wave localization is a peculiar property of random media that completely block wave propagation due to multiple scattering, thus inducing a surprising phase transition, for example, in optical or acoustic transparency or electrical conductivity. When localized, waves remain confined in space until dissipated.

More than two decades have passed since the phenomenon of wave localization was explored for propagation of electromagnetic (EM) waves in random media. During this period, a great body of literature has been generated[1]. And the interest in the subject continues to grow even further in recent years[2,3,4,5,6,7,8,9]. Despite the efforts, however, some important problems still remain unsolved.

The first issue is with the way in which the localization effect is investigated. To date, claims of localization have been based on observations of the exponential decay of waves as they propagate *through* disordered media[7]. That is, in most previous experimental or theoretical studies, the apparatus was set up in such a manner that waves were transmitted at one end of a scattering ensemble, then the scattered waves were recorded on the other end to measure the transmission through the sample. The results were then compared with the theory to infer the localization effect. In this method, it is quite plausible that other effects such as reflection and deflection due to the presence of boundaries may also attenuate waves, resulting in a similar decay in transmission and obscuring the data interpretation. Therefore it is desirable to look for a unique feature which can differentiate localization from other effects; the inability to discriminate the localization effect from other effects has caused significant debate in the literature[8].

Second, although it has been suggested a while ago that the regions of localized states coincide with the positions of the gaps, the relation between localization in random media and bandgaps of the corresponding regular systems is still inconclusive[4,10].

Third, it has been the prevailing view over the past twenty years that all EM waves are localized in two dimensions (2D) for any given amount of disorder, following the scaling analysis of electronic systems[11]. Recently, there is an intensive debate on this view from new experiments[12,13], as reviewed in[14]. Since localization in electronic and EM systems has the same physical origin, it is therefore imperative to re-look at the view that all EM waves are always localized in 2D random systems. This task may be difficult, due to the obvious limitation of the finite sample size for either numerical or experimental workers, but at least one may examine whether the phenomenon of localization has been explored in a proper way in the past.

With this Letter, we wish to shed new light on these questions. Here, we present a rigorous study of EM wave scattering and propagation in media containing many dielectric cylinders. The approach is based upon the self-consistent theory of multiple scattering[15] and has been used previously to study acoustic localization in liquid media[9] and acoustic attenuation by rigid cylinders in air[16]. Wave propagation is expressed by a set of coupled exact equations and is solved rigorously. We show that wave localization can be achieved in ranges of frequencies, coincident with yet wider than the complete bandgap. For the phenomenon of wave localization, we compare two scenarios by analogy with the acoustic case[17]: (1) the traditional setup of probing localization both numerically and experimnentally, as stated in, e. g. Ref.[4,7], that is, wave propagating through the array of cylinders, and (2) wave transmitted from a source located inside the ensemble. We show that within complete band gaps, results from the two scenarios are similar, whereas there is a distinct qualitative difference outside the gap. Moreover, when localized, not only are waves confined near the transmitting source but a unique collective phenomenon emerges, illustrated by a phase diagram in analogy to the acoustic system[9].

The system considered here is similar to what has been presented in[4]. Assume that N uniform dielectric cylinders of radius a are placed in parallel in a uniform medium, perpendicular to the $x - y$ plane. The arrangement can be either random or regular. For brevity, we only consider the case of the E-polarization, i. e. the E-field parallel to the z-direction. The qualitative features for both E- and H-polarizations are similar. The scatter-

ing and propagation of EM waves can be solved by using the exact formulation of Twersky[15]. While the details can be found in[16], here we brief the main procedures. A unit pulsating line source transmitting monochromatic waves is placed at a certain position. The scattered wave from each cylinder is a response to the total incident wave, which is composed of the direct contribution from the source and the multiply scattered waves from each of the other cylinders. The response function of a single cylinder is readily obtained in the form of the partial waves by invoking the usual boundary conditions across the cylinder surface. The total wave (E) at any space point is the sum of the direct wave (E_0) from the transmitting source and the scattered wave from all the cylinders. The normalized field is defined as $T \equiv E/E_0$; thus the trivial geometrical spreading effect is eliminated.
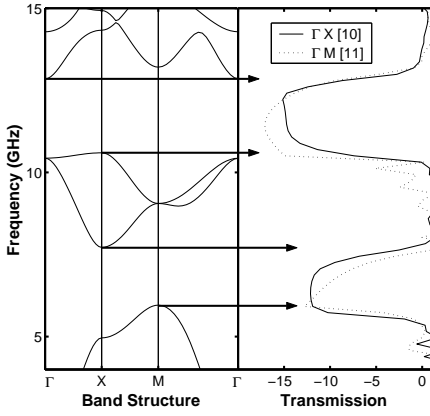

FIG. 1: Left panel: The band structures computed by the plane wave expansion method. Right panel: Here is shown the normalized transmission $\log_{10} |T|^2$ versus frequency; the solid line refers to the result from the [10] direction propagation, and the dotted line to that from the [11] direction propagation lines.

In line with[4], the following parameters are used in the computation. The ratio of the dielectric constant between the cylinders and the hosting medium is 10; the dielectric constant of the medium is taken as one. The filling factor β, the fraction of area occupied by the cylinders per unit area, is 0.28. The radius a of the cylinders is 0.38 cm. The lattice constant d of the corresponding square lattice array of the cylinders is thus about 1.28 cm $(d = a\sqrt{\pi/\beta})$. For convenience, we scale all lengths by the lattice constant d. The computation is continued until the convergence is reached.

First, in Fig. 1 we show the band structure of the corresponding square lattice arrangement of the cylinders, obtained by the plane wave method. The wave transmission in two symmetric directions is also shown. Two complete bandgap regions are identified and are consistent with the highly attenuated regions in the transmission computation. These results are also consistent with that in Fig. 4 of[4], thereby verifying our numerical scheme.

To investigate wave localization, two situations are considered and compared: (1) wave propagating through the array of cylinders, labeled hereafter as the 'Outside' situation that imitates the traditional experimental[5] and theoretical setups[4,6], and (2) wave transmitted from a source located inside the ensemble, labeled hereafter as the 'Inside' situation. Both cases are illustrated by Fig. 2. For the 'Outside' case, all cylinders are randomly or regularly placed within a rectangular area with length L and width W. The transmitter and receiver are located at some distance from the two opposite sides of the scattering area. For the 'Inside' situation, all cylinders are placed either completely randomly or regularly within a circle of radius L with the source located at the center and the receiver located outside the scattering cloud.
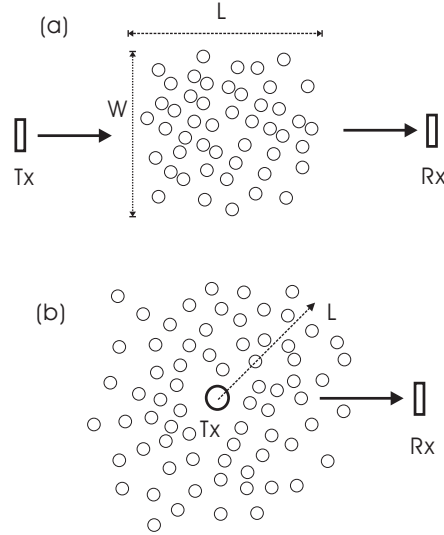

FIG. 2: (a) The 'Outside' case: Electromagnetic propagation through a cloud of dielectric cylinders. (b) The 'Inside' case: Electromagnetic transmission from a line source located inside the array of dielectric cylinders.

The frequency response of the averaged logarithmic transmission is presented in Fig. 3 for both 'Inside' and 'Outside' scenarios. Here we see that the disorder somewhat tends to enhance transmission within the bandgaps for both scenarios, while obviously reduces the transmission for all frequencies outside the gaps in the 'Outside' case. For the 'Inside' situation, however, the reduction for regions outside the gaps is not generally obvious, and is only seen near the gap edges. It has been suggested in the literature that the transmission reduction in the 'Outside' scenario indicates wave localization. For example, the authors in[4,6] computed the transmission in the context of the 'Outside' scenario, and subsequently obtained the localization length for all frequencies. We find that this approach towards localization may not be appropriate. The reasons follow.

If the transmission reduction in the 'Outside' scenario is only caused by the localization effect, it will be implied that the random system only supports localized states. Then waves will not be allowed to propagate not only through but also inside the system. Therefore we would

expect the transmission to follow an exponential decay with increasing sample size for both 'Inside' and 'Outside' setups.
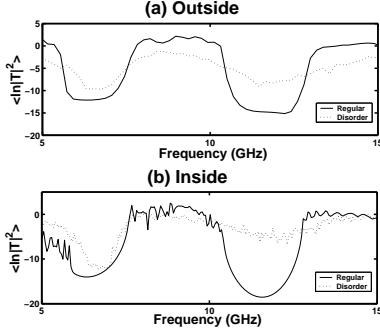

FIG. 3: Transmission versus frequency for both random and regular arrays of cylinders: (a) the 'Outside' case with W = 6 and L = 10; and (b) the 'Inside' case with L = 10. Please refer to Fig. 2 and the text for the explanation about the 'Outside' and 'Inside' cases.

Fig. 4 presents the results for the random ensemble averaged transmission and its fluctuation as a function of the sample size at two frequencies. The sample size is varied by adjusting the number of the cylinders. For the 'Outside' case, we have done the following to remove the effect of the width W. With a fixed sample size (i. e. the length L), we plot the transmission versus width. We find that the transmission is very nicely saturating to a certain value in an exponential manner. We have done for several lengths, and obtained the corresponding saturated value for each length. Then we plot these values versus sample lengths. As an example, the results for 8.64 GHz are shown in Fig. 4 (e) and (f). For 6.54 GHz, the localization is strong, the width effect diminishes very quickly when the width increases. Here the plot for 6.54 GHz has width 26 in the 'Outside' scenario. Note that the width should not be started at a value too close to zero; otherwise the variance will be too large, making the results unstable. The average has been taken for 500 configuration to ensure the stability.

A few important features are discovered. For the frequency of 6.54 GHz (within the first gap), the transmission decays exponentially with the sample size for both 'Outside' and 'Inside' situations with almost the same slop of -1.35, suggesting that at this frequency, waves are localized. And inside the localization regime, the absolute value of the transmission fluctuation is small, as expected from an earlier work[9]. Here we see that within the localization regime, wave localization can be indeed observed in both 'Outside' and 'Inside' scenarios.

For 8.64 GHz (between the first and the second gaps), the 'Outside' and 'Inside' scenarios differ significantly. For the 'Outside' case the transmission decreases exponentially with a slop of -0.0612 along the path. If this exponential decay is caused by localization, then we should also observe the exponential decay for the same sample size (L) in the 'Inside' scenario. The result in the center panel of Fig. 4 clearly does not support this point of view. Instead, Fig. 4 tends to indicate that waves are not yet localized at 8.64 GHz in the 'Inside' scenario. The fact that the exponential decay only occurs in one scenario but not in the other for the same sample size (L) is itself intriguing and important. Therefore we may conclude that the 'Outside' scenario is inappropriate in isolating the localization effect, and it would be a mistake to interpret the exponential decay or transmission reduction shown in the 'Outside' situation as a conclusive indication of wave localization. Furthermore, as at this frequency, waves are not yet localized in the 'Inside' case and they have a weaker exponential decay in the 'Outside' case, the transmission will be more sensitive to the arrangement of the cylinders. Therefore the fluctuation at this frequency is stronger than that at 6.54 GHz. However, the ratio between the fluctuation and the transmission at 8.64 GHz can be smaller than that at 6.54 GHz.
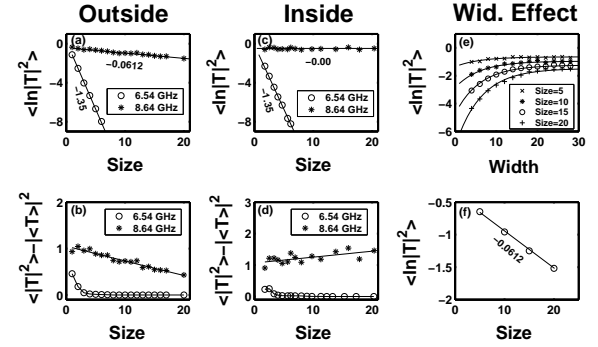

FIG. 4: The averaged logarithmic transmission and its fluctuation versus the sample size for two frequencies: one is within the first bandgap and the other is above the first but below the second gap. The left and center panels refer to the 'Outside' and 'Inside' cases respectively. The estimated slops for the transmission are indicated in the figure. The right panel shows the effect of width W and the plot of the transmission versus length L at the extrapolated infinite width (see the text).

To this end, a few notes are appropriate. We have also examined other frequencies in general and two in particular: one is within the second gap and the other is above the second gap. The results are very similar to that shown in Fig. 4. For brevity, we will not show the results here. From Fig. 3, the fact that the transmission reduction occurs not only within but also outside the gaps (at areas around the edges of the gaps) indicates that the localized regions are coincident with the complete bandgaps, and these regions seem wider than the gaps. Our results show that although the disorders may block waves from propagation *through* the medium, but they may not yet localize the waves inside a 2D system.

Now we discuss a unique feature of EM wave localization. The energy flow of EM waves is $\vec{J} \sim \vec{E} \times \vec{H}$. By invoking the Maxwell equations to relate the electrical and magnetic fields, we can derive that the time averaged energy flow is $<\vec{J}>_t \equiv \frac{1}{T}\int_0^T dt \vec{J} \sim |\vec{E}|^2 \nabla \theta$, where

the electrical field is written as $\vec{E} = \vec{e}_E|\vec{E}|e^{i\theta}$, with $\vec{e}_E$ denoting the direction, $|\vec{E}|$ and θ being the amplitude and the phase respectively. It is clear that when θ is constant, at least by spatial domains, while $|\vec{E}| \neq 0$, the flow would come to a stop and the energy will be localized or stored in the space. We assign a unit phase vector, $\vec{u} = \cos\theta_i \vec{e}_x + \sin\theta_i \vec{e}_y$ to the oscillation phase θ_i of the dipoles. Here $\vec{e}_x$ and $\vec{e}_y$ are unit vectors in the x and y directions respectively. These phase vectors are represented by a phase diagram in the $x - y$ plane.
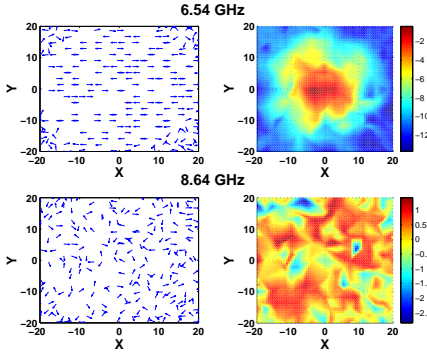

FIG. 5: The phase diagram and spatial distribution of electromagnetic energy for two frequencies for one random configuration. Left panel: the phase diagram for the phase vectors defined in the text; here the phase of the direct field E_0 is set to zero. Right panel: the energy spatial distribution.

In Fig. 5, the two-dimensional spatial distribution of EM energy ($\sim |E/E_0|^2$) and the phase vectors of the E-field are plotted for the two frequencies discussed in Fig. 4. The phase vectors are located randomly in the $x - y$ plane but to avoid the positions of the cylinders. The 'Inside' scenario is considered. Here we clearly see that for 6.54 GHz, the energy is mainly confined near the source, consistent with Fig. 4. The phase vectors are orderly oriented. These fully comply with the above general discussion. Therefore at this frequency, EM wave is indeed localized. When we increasingly add an imaginary part to the dielectric constant, the ordered orientation of the phase vectors will disappear, confirming that the phase coherence is a unique feature of EM wave localization. We note from Fig. 5 that near the sample boundary, the phase vectors start to point to different directions. This is because the numerical simulation is carried out for a finite sample size. For a finite system, the energy can leak out at the boundary, resulting in disappearance of the phase coherence. When enlarging the sample size, we observe that the area showing the perfect phase coherence will increase. At 8.64 GHz, however, there is no ordering in the phase vectors $\vec{u}(\vec{r})$. The phase vectors point to various directions. The energy distribution is extended in the $x - y$ plane, and no EM wave localization appears, in agreement with what has been described for Fig. 4.

In summary, we have examined some fundamental problems of EM wave localization in 2D. Although it may be still hard to conclude that extended waves are possible in 2D random media, as limited by the finite sample size, the present results do indicate that the traditional method may be unable to isolate the localization effect. It is also shown that the localization region is related to and seems to be wider than the complete bandgaps. When localized, not only are waves confined near the transmitting source but a unique collective phenomenon emerges.

* Electronic address: zhen@phy.ncu.edu.tw
[1] For a review, please refer to S. John, Phys. Today **40**, 32 (1991); *Scattering and localization of classical waves in random media*, edited by P. Sheng (World Scientific, Singapore, 1990); A. Lagendijk and B. A. van Tiggelen, Phys. Rep. **270**, 143 (1996).
[2] Y. Kuga, A. Ishimaru, J. Opt. Soc. Am. A1, 831 (1984); M. P. van Albada, A. Lagendijk, Phys. Rev. Lett. 55, 2692 (1985); P. E. Wolf, G. Maret, Phys. Rev. Lett. 55, 2696 (1985); A. Tourin, et al., Phys. Rev. Lett. 79, 3637 (1997); M. Torres, J. P. Adrados, F. R. Montero de Espinosa, Nature **398**, 114 (1999).
[3] M. Rusek and A. Orlowski, Phys. Rev. E**51**, R2763 (1995).
[4] M. M. Sigalas, C. M. Soukoulis, C.-T. Chan, and D. Turner, Phys. Rev. B**53**, 8340 (1996).
[5] D. S. Wiersma, P. Bartolini, A. Lagendijk, and R. Roghini, Nature **390**, 671 (1997).
[6] A. A. Asatryan, et al. Phys. Rev. B**57**, 13535 (1998).
[7] A. A. Chabanov, M. Stoytchev, and A. Z. Genack, Nature **404**, 850 (2000).
[8] F. Scheffold, R. Lenke, R. Tweer, and G. Maret, Nature **398**, 206 (1999); D. Wiersma, et al. Nature, **398**, 207 (1999).
[9] E. Hoskinson and Z. Ye, Phys. Rev. Lett. **83**, 2734 (1999).
[10] C. M. Soukoulis, S. Datta, and E. N. Economou, Phys. Rev. B**49**, 3800 (1994).
[11] E. Abrahams, P. W. Anderson, D. C. Licciardello, and T. V. Ramakrishnan, Phys. Rev. Lett. **42**, 673 (1979).
[12] V. M. Pudalov, M. D'Iorio, S. V. Kravchenko, and J. W. Campbell, Phys. Rev. Lett. **70**, 1866 (1993).
[13] A. A. Shashkin, V. T. Dolgopolov and G. V. Kravchenko, Phys. Rev. B**49**, 14486 (1994).
[14] E. Abrahams, S. V. Kravchenko, and M. P. Sarachik, Rev. Mod. Phys. **73**, 251 (2001).
[15] V. Twersky, J. Acoust. Soc. Am. **24**, 42 (1951).
[16] Y.-Y. Chen and Z. Ye, Phys. Rev. Lett. **87**, 184301 (2001).
[17] Y.-Y. Chen and Z. Ye, Phys. Rev. E**65**, 056612 (2002).